

Figure 3.3 - The three configurations are equivalent if the four particles are indistinguishable amongst themselves.

Each of the n states can be associated with A, B, C, etc. (that is, to each or more of the m particles) and since a single particle can occupy each time a different state (and other particles other states), m times, the possible combinations C are $n \times n \times n \times \dots \times n$ (m factors equal to n).

$$C = n^m.$$

We could also be convinced, observing for example *Figure 3.4* where it is assumed that $n=5$ (it looks like a musical stave...) and $m=2$ particles (therefore $5^2=25$ combinations):



Figure 3.4 – Beyond the 25th beat the preceding configurations are repeated because A and B are indistinguishable. Within the range of the 25 possible configurations some are more favored because they appear more frequently: for example 6 and 22, 9 and 25, etc. The unoccupied states are identified by a circle. As is fair to expect, configuration 1 is least favored.



We can arrive at the same result with a more practical method, suitable also for very large values of n and m , which we will use as follows.

It consists of a tabular method stolen from Combinatorial Analysis where for n and m equal to various units it avoids the need to write hundreds or thousands of key strokes as used above.

Let us take two rows and as many columns as there are states, thereby obtaining a grid: in *Figure 3.5*, to verify what has been said above, we have taken 2 rows and 5 columns ($n=2; m=5$).

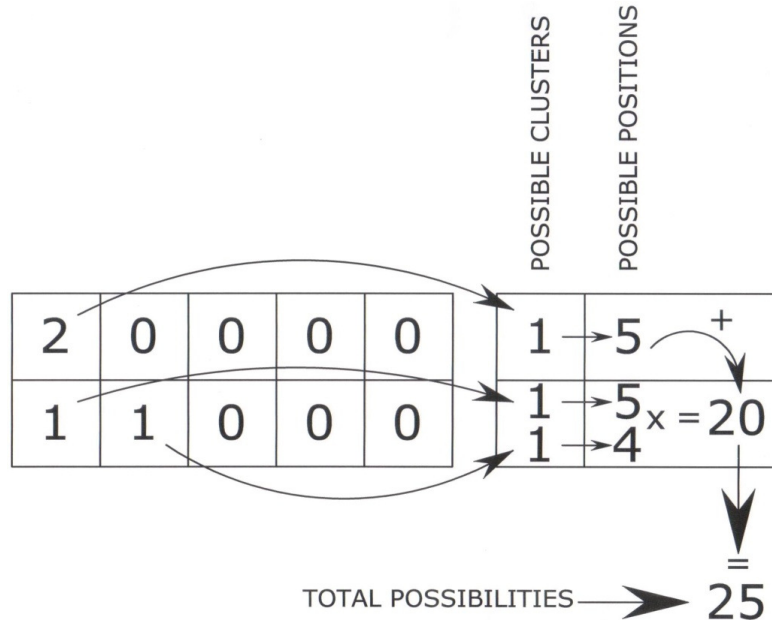


Figure 3.5.- With this grid we obtain the number of possible configurations.

To further demonstrate we will build a grid for $n=5$ and $m=4$, as in *Figure 3.6*, where there are sufficient rows to progressively expose the number of particles (from 4 to 1 in the first box of the first column of the occupancy numbers) and there are n columns.



		OCCUPATION NUMBER					POSSIBILITY	PROBABILITY
①		4	0	0	0	0	5	0,8%
②		3	1	0	0	0	80	12,8%
③		2	2	0	0	0	120	19,2%
④		2	1	1	0	0	360	57,6%
⑤		1	1	1	1	0	60	9,6%
						POSSIBILITY OF CONFIGURATION TOTAL	PROBABILITY TOTAL	
						625	100%	

Figure 3.6 - Since $5^4 = 625$, there are 625 possible combinations; the relative probabilities are listed in the last column: note the asymmetry.



It is necessary to observe that in the figure the table of numbers of occupancy reminds us, not by chance, of Tartaglia's Triangle, while the Boltzmann type diagram that can be associated, shown in *Figure 3.7*, takes on an almost familiar shape.

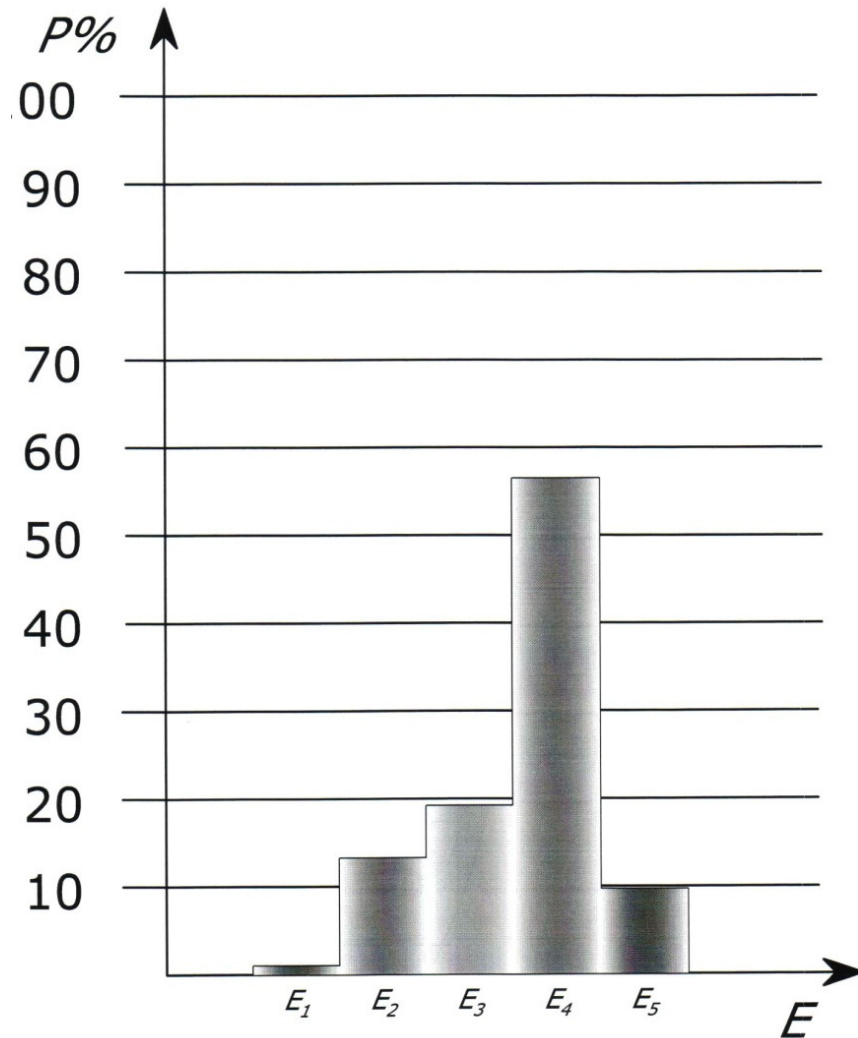


Figure 3.7 - Graphical representation of Figure 6.2; the bars are asymmetric.